

Investigate Rules of Games

Practice 1

Suggested solution

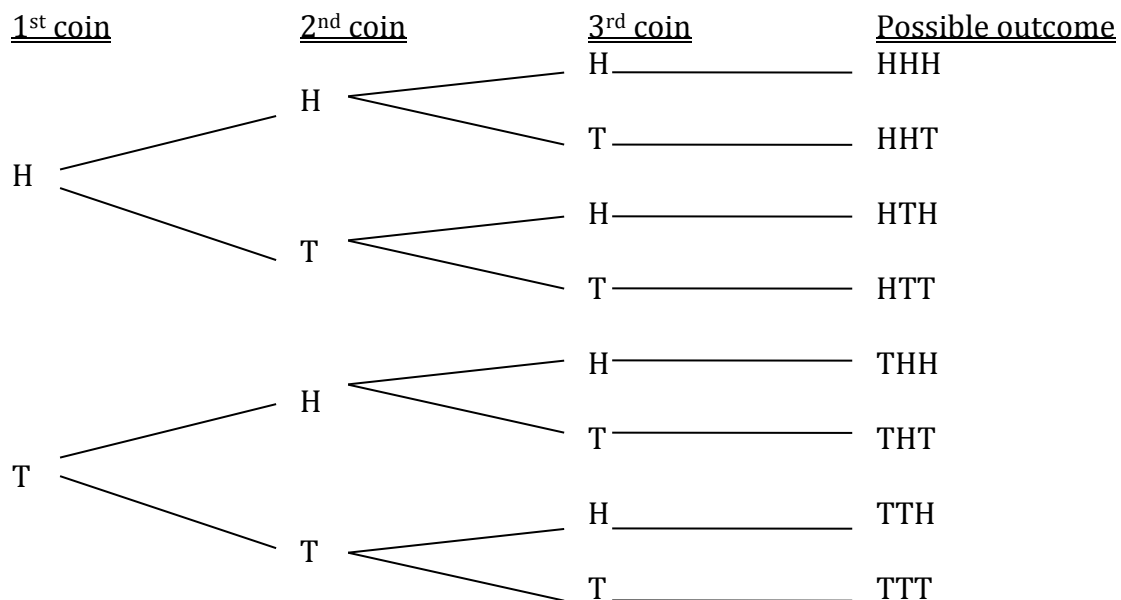
Objective of the task:

To investigate and generate rules for the probabilities of different events if you toss three coins at the same time.

Solution:

Step 1: I am going to use a tree diagram to find all possible outcomes when tossing three coins at the same time.

- Let “H” stands for “head” of the coin (the flower/ head/ picture side)
- Let “T” stands for “tail” of the coin (the number side, which shows the value of the coin)



Step 2: I am going to do an experiment (toss the 3 coins many times) to find the experimental probabilities of each event.

Events of tossing 3 coins	Tally	Frequency	Experimental Probability $\frac{\text{number of times the event occurred}}{\text{total number of trials}}$
HHH		67	$\frac{67}{500} = 0.134$
HHT		68	$\frac{68}{500} = 0.136$
HTH		62	$\frac{62}{500} = 0.124$
HTT		66	$\frac{66}{500} = 0.132$
THH		56	$\frac{56}{500} = 0.112$
THT		52	$\frac{52}{500} = 0.104$
TTH		65	$\frac{65}{500} = 0.130$
TTT		64	$\frac{64}{500} = 0.128$
Total:		500	1

I have tossed the three coins 500 times and recorded the above results.

Step 3: I am going to find and explain the rules and patterns from the experimental results

From the above experimental results and probabilities, I find out that:

- (i) there are 8 possible outcomes and each outcomes have more or less equal chance to happen or has same probability,
- (ii) the total of the experimental probabilities of all the outcomes is 1,
- (iii) if I don't care about the order of the heads and tails, I can regroup the events as:
 - a. getting all 3 heads
 - b. getting only 2 heads
 - c. getting only 1 head
 - d. getting no head at all

Then, the rules and patterns will change as:

Events of tossing 3 coins	Frequency	Experimental Probability $= \frac{\text{number of times the event occurred}}{\text{total number of trials}}$
getting all 3 heads (HHH)	67	$\frac{67}{500} = 0.134$
getting 2 heads (HHT/HTH/THH)	68+62+56=186	$\frac{186}{500} = 0.372$
getting only 1 head (HTT/HTT/TTH)	66+52+65=183	$\frac{183}{500} = 0.366$
getting no head at all (TTT)	64	$\frac{66}{500} = 0.128$
Total	500	1

- a) There are the least chance to get all 3 heads or no head,
Which is $P(\text{all 3 heads}) = \frac{67}{500} = 0.134$ or $P(\text{no head}) = \frac{64}{500} = 0.128$
- b) There will be more chance to get only 2 heads (HHT/HTH/THH) or only 1 head (HTT/THT/TTH),
Which is $P(2 \text{ heads}) = \frac{68+62+56}{500} = \frac{186}{500} = 0.372$ or $P(1 \text{ head}) = \frac{66+52+65}{500} = \frac{183}{500} = 0.366$
- c) The chance of getting all heads or getting no head will be around $\frac{1}{3}$ of the chance of getting 2 heads or 1 head,
Which is $\frac{P(\text{all heads})}{P(2 \text{ heads})} = \frac{0.134}{0.372} \approx \frac{1}{2.78} \approx \frac{1}{3}$ or $\frac{P(\text{no heads})}{P(1 \text{ head})} = \frac{0.128}{0.366} \approx \frac{1}{2.86} \approx \frac{1}{3}$
- d) All of the 4 cases added together, the probabilities will be 1 again.

Step 4: Test/Justify my rules/patterns are correct by finding the theoretical probabilities

Events of tossing 3 coins	Theoretical Probability $= \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$	Experimental Probability $= \frac{\text{number of times the event occurred}}{\text{total number of trials}}$
HHH	$\frac{1}{8} = 0.125$	0.134
HHT	$\frac{1}{8} = 0.125$	0.136
HTH	$\frac{1}{8} = 0.125$	0.124
HTT	$\frac{1}{8} = 0.125$	0.132
THH	$\frac{1}{8} = 0.125$	0.112
THT	$\frac{1}{8} = 0.125$	0.104
TTH	$\frac{1}{8} = 0.125$	0.130
TTT	$\frac{1}{8} = 0.125$	0.128
Total	1	1

By comparing the experimental probabilities and the theoretical probabilities:

(i) it is very clear that all the 8 events should have the same probability that matches with my findings. Even though the experimental probabilities are not exactly the same, but I trust that if I have done many more times of the experiment, it might be even closer to the theoretical ones.

(ii) The total of the probabilities of all events is 1, no matter it is theoretical or experimental probabilities, that prove my rules and patterns are correct.

(iii) Let me regroup and list the probabilities again:

Events of tossing 3 coins	Theoretical Probability $= \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$	Experimental Probability $= \frac{\text{number of times the event occurred}}{\text{total number of trials}}$
getting all 3 heads (HHH)	$\frac{1}{8} = 0.125$	0.134
getting 2 heads (HHT/HTH/THH)	$\frac{3}{8} = 0.375$	0.372
getting only 1 head (HTT/HTT/TTH)	$\frac{3}{8} = 0.375$	0.366
getting no head at all (TTT)	$\frac{1}{8} = 0.125$	0.128
Total	1	1

From the above table, we can easily see that the theoretical probabilities are very close to the experimental probabilities of the same or corresponding events.

When we compare the probability of getting all heads or getting no head will be around $\frac{1}{3}$ of the chance of getting 2 heads or 1 head with the theoretical ones as follow:

$$\frac{P(3 \text{ heads})}{P(2 \text{ heads})} = \frac{0.125}{0.375} = \frac{1}{3} \quad \text{and} \quad \frac{P(\text{no head})}{P(1 \text{ head})} = \frac{0.125}{0.375} = \frac{1}{3}$$

which again matches with the patterns shown in step 3 (iiic).

As a conclusion, the rules and pattern for the probabilities of different events of tossing 3 coins at the same time are:

- (i) There are 8 possible outcomes (HHH, HHT, HTH, HTT, THH, THT, TTH, TTT)
- (ii) The eight different events have the same probability
- (iii) All the eight events' probabilities added up to be 1.

If we regrouped the events as the follow 4 types:

- a. getting all 3 heads (HHH)
- b. getting only 2 heads (HHT, HTH, THH)
- c. getting only 1 head (HTT, TTH, THT)
- d. getting no head at all (TTT)

$$P(\text{all heads}) = P(\text{no head}) = 0.125$$

$$P(2 \text{ heads}) = P(1 \text{ head}) = 0.375$$

Therefore, getting $P(\text{all heads})$ or $P(\text{no head})$ will be $\frac{1}{3}$ of getting $P(2 \text{ heads})$ or $P(1 \text{ head})$.